

Wave height distributions and spectral properties in the nearshore region



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**Pacific coast, Canada
<1 km from shore:**

- **Pressure sensors,
long-term record**



Public Safety
Canada

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Natural Sciences and Engineering
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Discovery Grant

Canada

Location

Pacific Rim
National Park,
Vancouver Island,
BC, Canada

- Harsh wave climate ($H_s > 8\text{m}$, near shore)
- Long fetch
- Swell

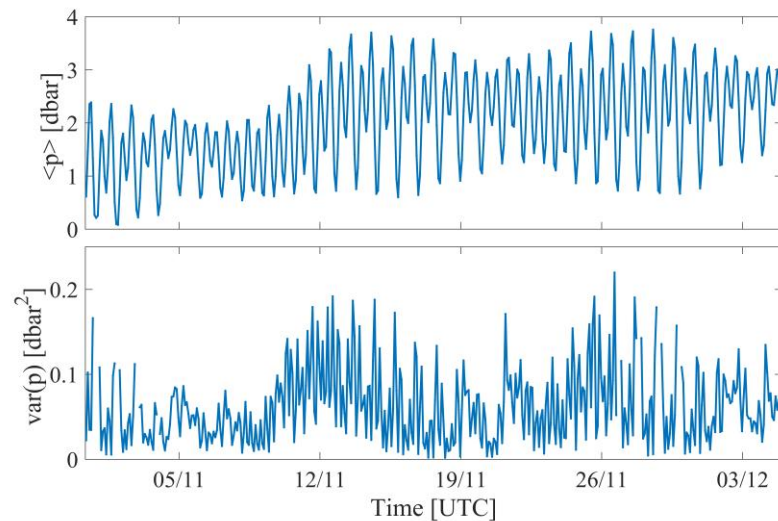
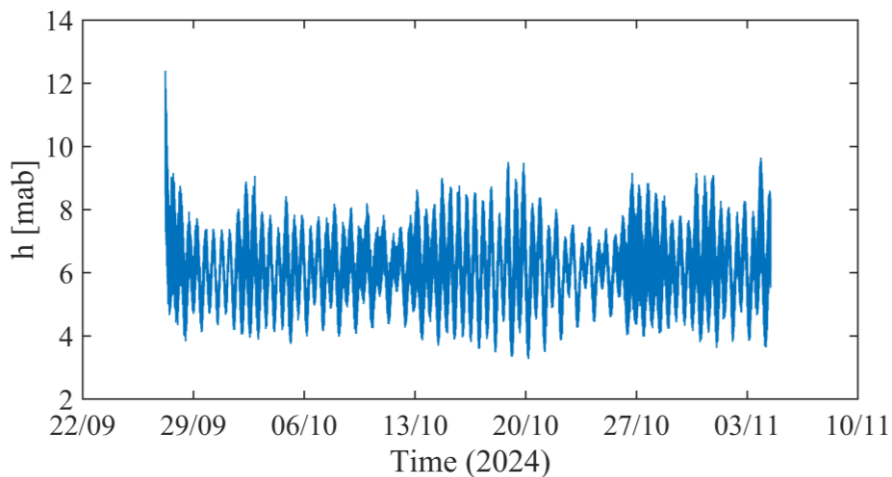


Observations: bottom-mounted pressure recorder (2Hz)

1 km offshore,
2 sites, D 8m, D= 6m



Inter-tidal zone



Total record length: 362 days

Pressure → surface elevation

$$\eta_H = \frac{p - p_{atm}}{\rho g} - z_0 \quad z_0: \text{mean water depth}$$

Hydrostatic

$$\eta_{SL} = \eta_H - \frac{z_0}{2g} \frac{\partial^2 \eta_H}{\partial t^2}$$

Shallow water, linear

$$\eta_{SNL} = \eta_{SL} - \frac{1}{g} \left[\left(\frac{\partial \eta_{SL}}{\partial t} \right)^2 + \eta_{SL} \frac{\partial^2 \eta_H}{\partial t^2} \right]$$

Shallow water, non-linear

- Applied locally in time
- Weakly dependent on low-pass filter



Contents lists available at [ScienceDirect](#)

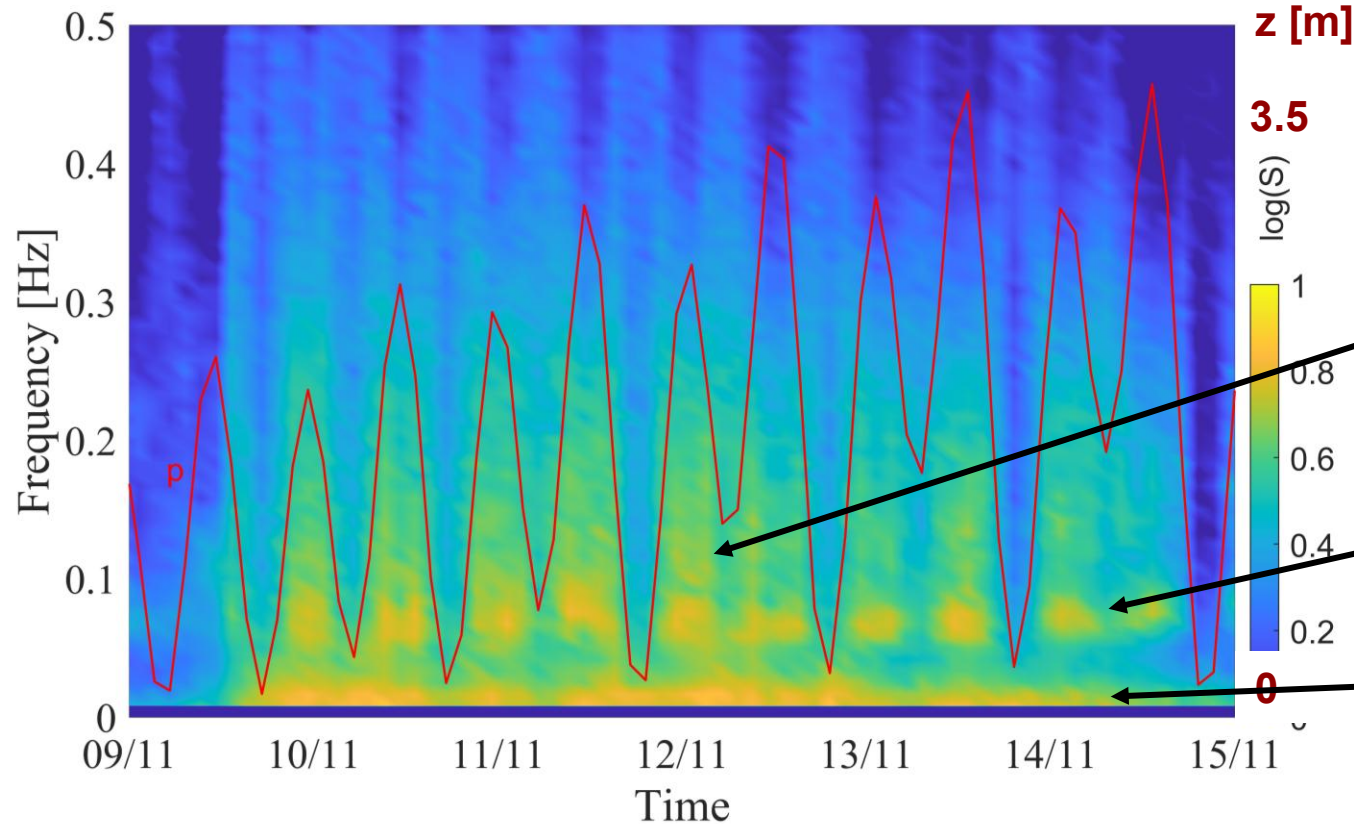
Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

A nonlinear weakly dispersive method for recovering the elevation of irrotational surface waves from pressure measurements

P. Bonneton^{a,*}, D. Lannes^b, K. Martins^c, H. Michallet^d

Spectral properties in varying water depth

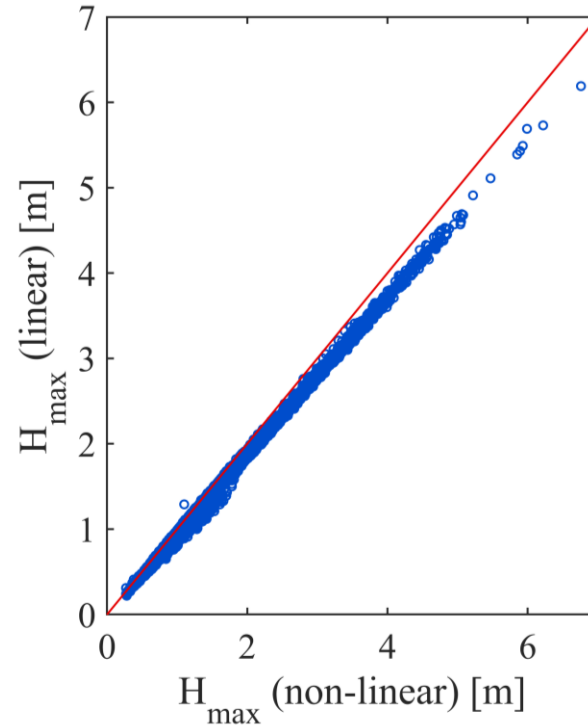
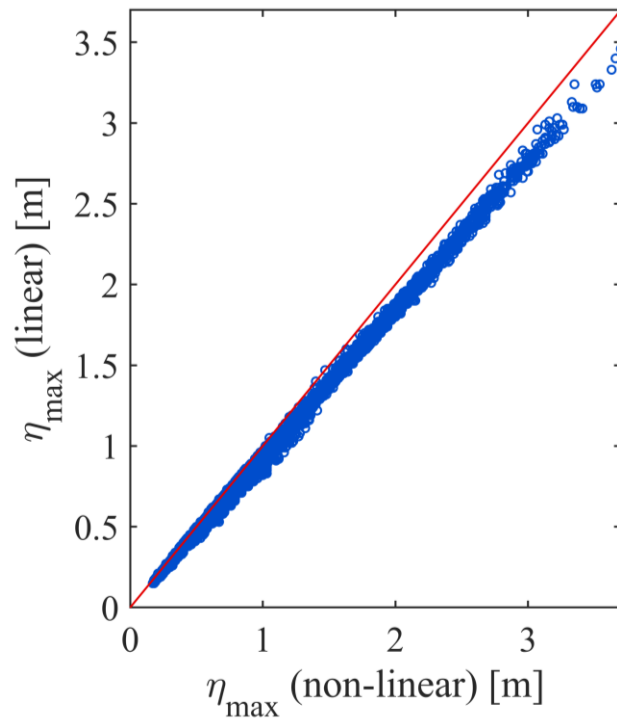


Higher harmonics
(triads)

Dominant wind
waves

Infra-gravity

Surface elevation retrieval – maximum height



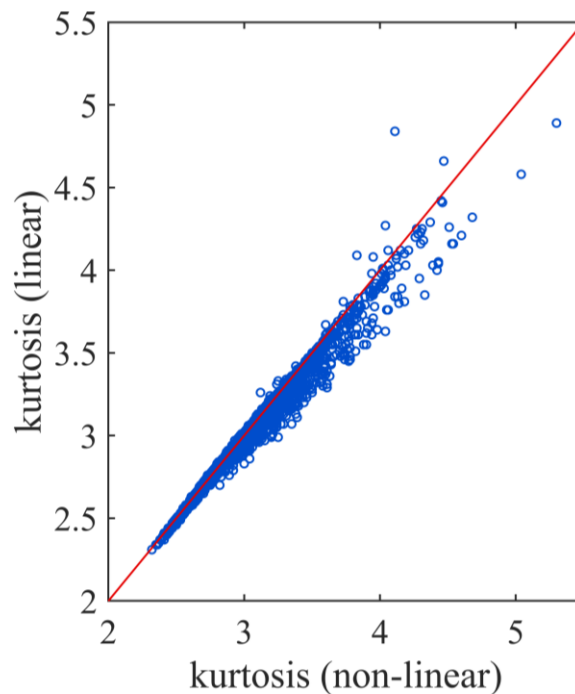
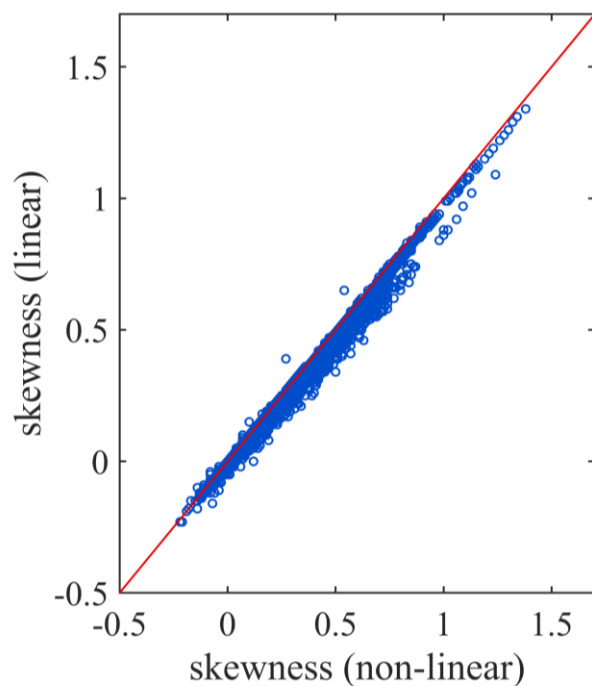
**Under-estimation
by linear method**

Crest height: 8.2%

Wave height: 7.3%

8705 hourly data

Surface elevation retrieval – higher moments



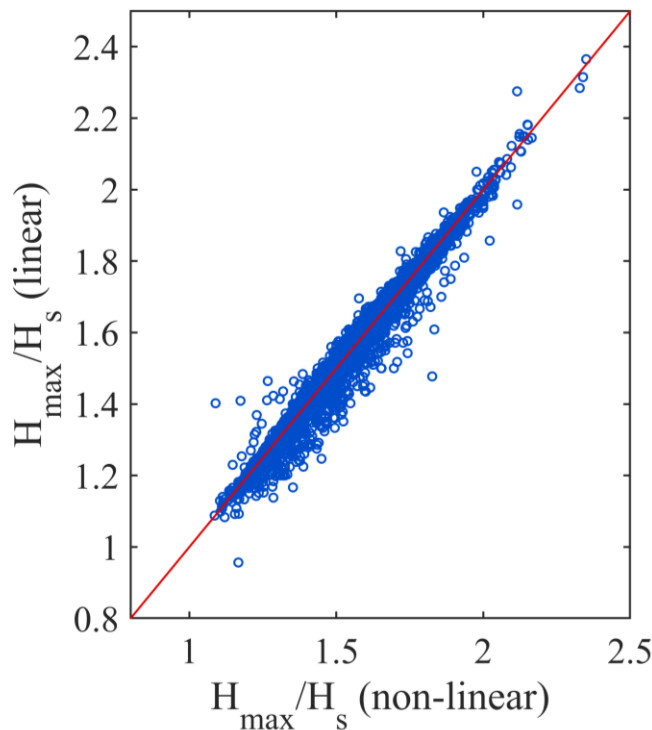
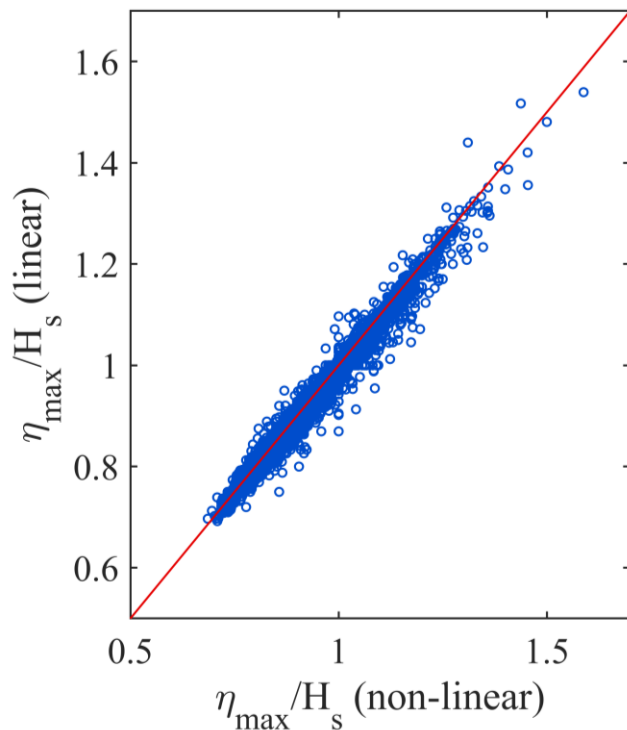
**Under-estimation
by linear methods**

Skewness: 4.0%

Kurtosis: 7.3%

8705 hourly data

Surface elevation retrieval – normalized maximum height



Only small offset by linear methods

Crest height: 3.0%
Wave height: 0.7%

Useful parameter

Wave nonlinearity: Ursell number

$$Ur = 4 \pi^2 \frac{H/D}{(kD)^2} \propto \frac{\textit{relative wave height}}{(\textit{relative water depth})^2}$$

Ur < 20: deep water, linear waves

20 < Ur < 50: intermediate nonlinearity

Ur > 50: shallow water, nonlinear waves

Presentation

Weibull distribution:

$$P\left(\frac{\eta}{H_s} > z\right) = \exp(-\beta z^\alpha)$$

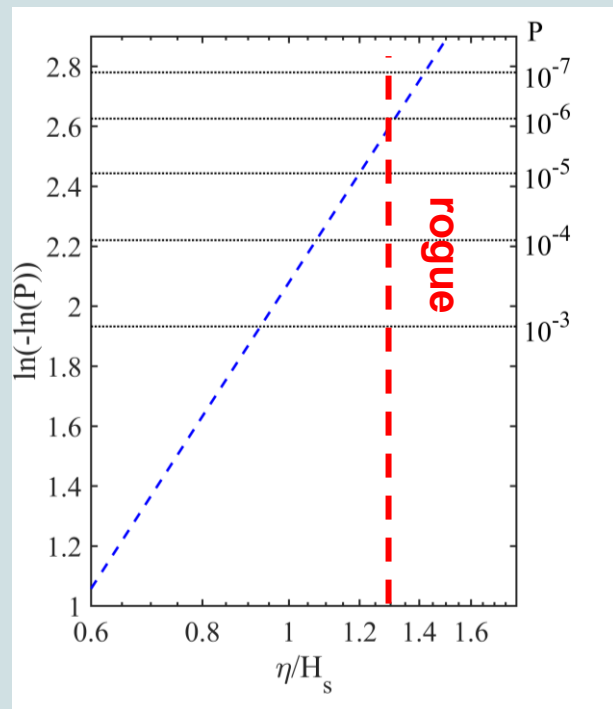


$$\ln(-\ln(P)) = \alpha \ln(z) + \ln(\beta)$$

Any Weibull distribution = straight line

α : slope, $\ln(\beta)$: y-value at $z=1$

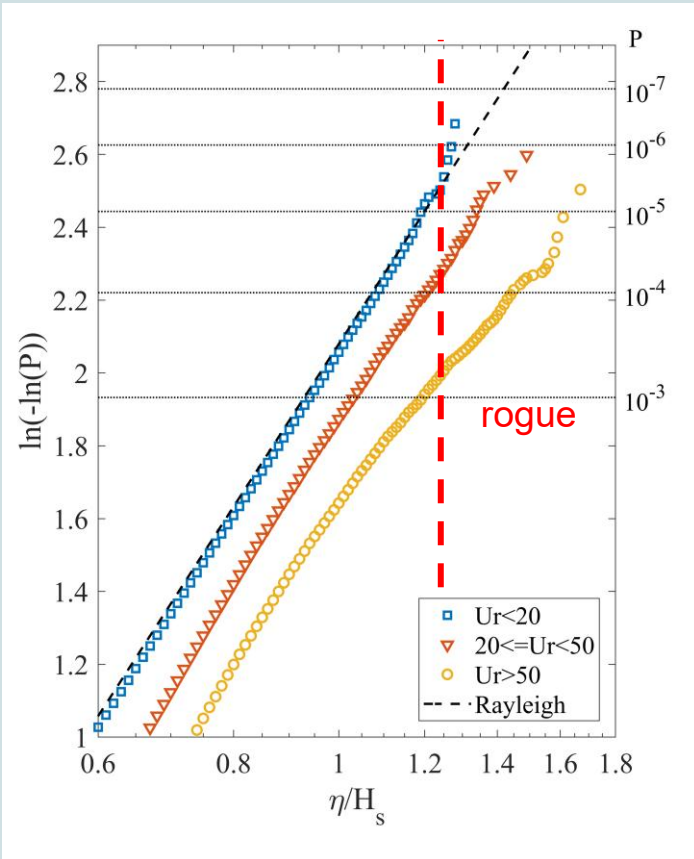
Note: Rayleigh distribution: $\alpha = 2$, $\beta = 8$



Probability →

Normalized height

Crest height distributions – wave nonlinearity



Probability →

Crests are taller than in Gaussian sea

Frequent rogue crests ($\eta/H_s > 1.25$)

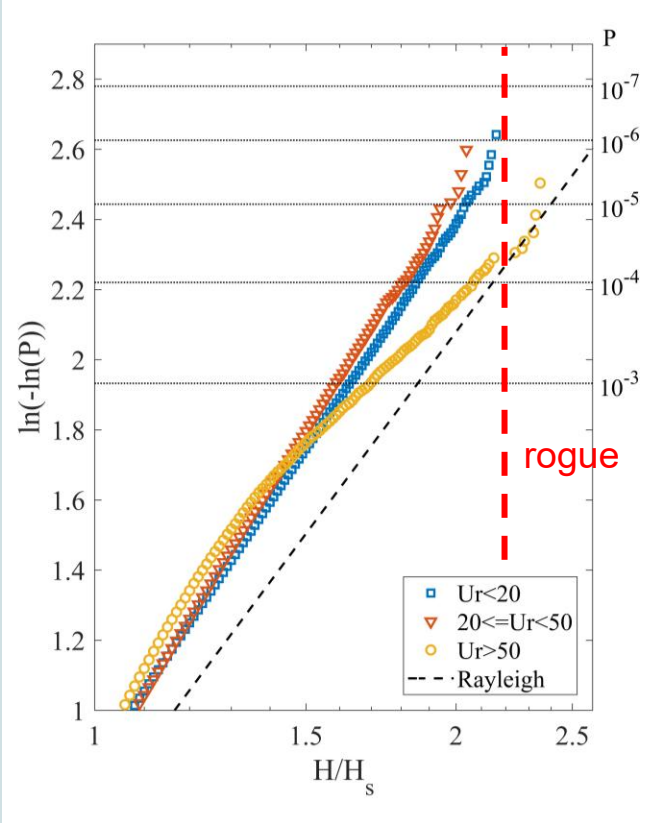
$Ur < 20$: Rayleigh distribution, $\alpha=2, \beta=8$

$Ur > 20$: Weibull distribution, $\alpha=2, \beta \ll 8$

$Ur > 50$: convex → extreme crests have increased probability (= true rogues)

Normalized crest height

Wave height distributions – wave nonlinearity



Normalized wave height

Probability →

Waves are smaller than in Gaussian sea

Very rare rogue waves ($H/H_s > 2.2$)

$U_r < 50$: Weibull distribution, $\alpha=2, \beta>2$

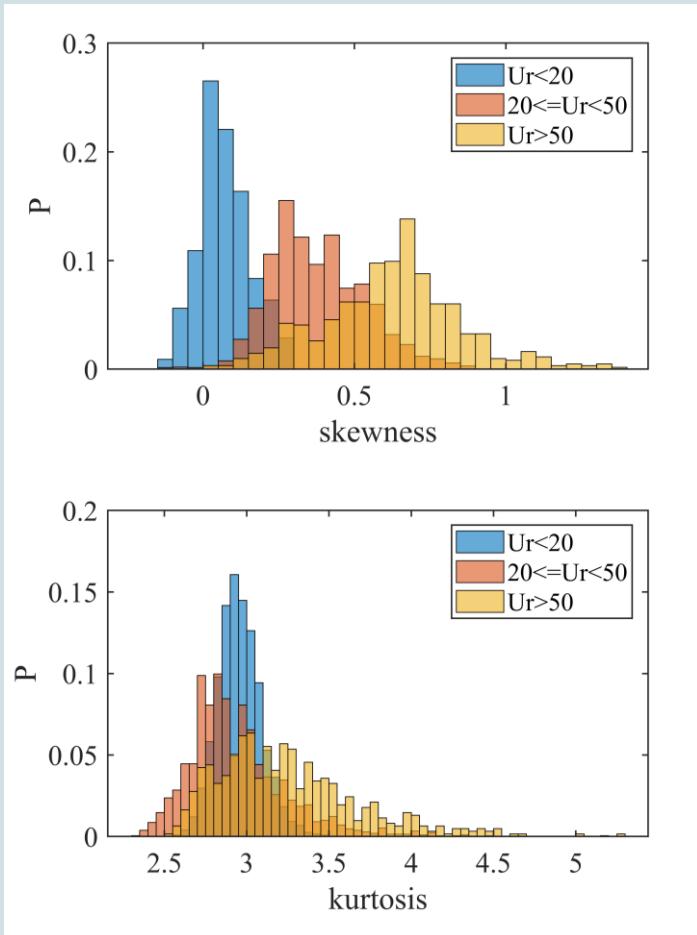
$U_r > 50$: Composite Weibull Distribution

$H/H_s < 1.5$: $\alpha=2.1$

$H/H_s > 1.5$: $\alpha=1.4$

→ Larger waves more likely

Spectral properties - nonlinearity



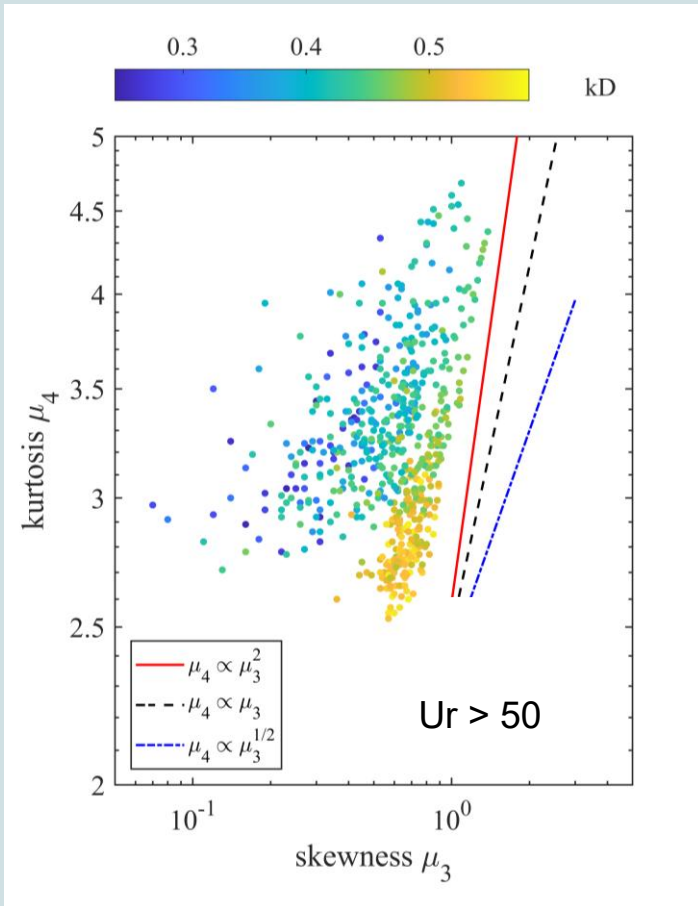
Skewness:

- **Positive \rightarrow Crest-trough asymmetry in shallow water**
- **Mean increases with nonlinearity**

Kurtosis:

- **Mean = 3 (Gaussian)**
- **Spread increases with nonlinearity**

Kurtosis - skewness



$$\mu_4 \propto \mu_3^n$$

2nd- order theory:

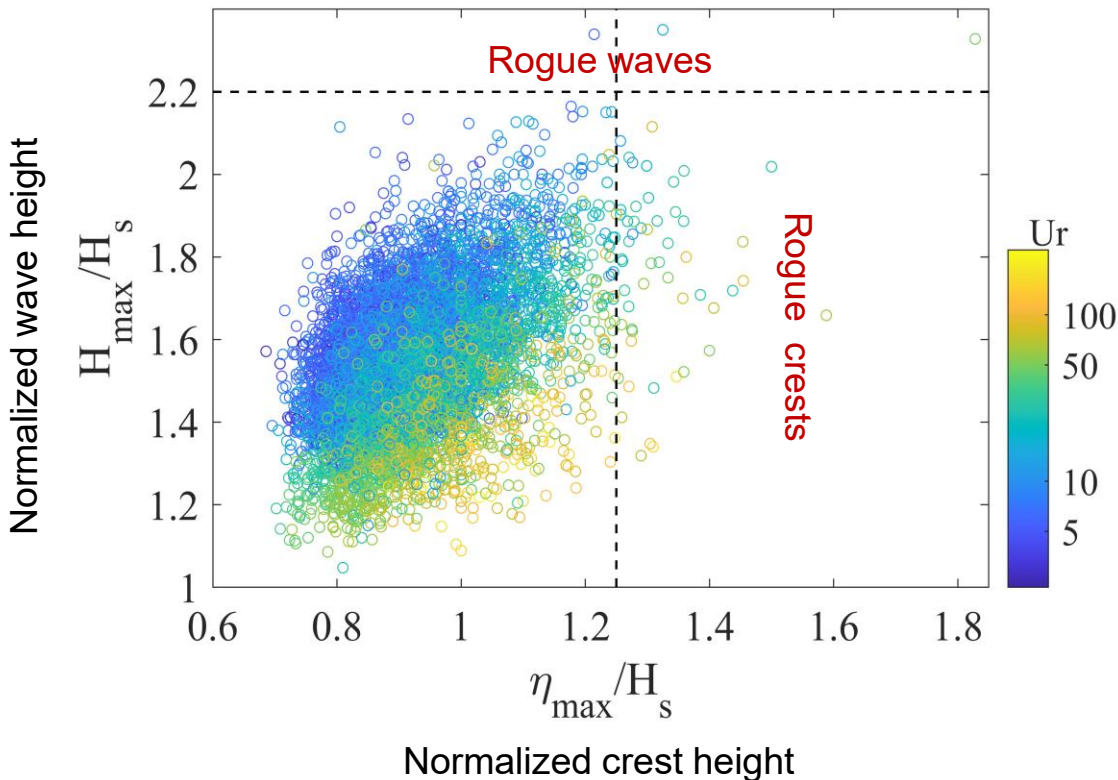
$n = 2$, also observed in wave tanks

Observations:

$n = 1$ for intermediate water depth $kD > 0.4$

$n < 1$ for very shallow water $kD < 0.4$

Maximum wave and crest height (hourly)



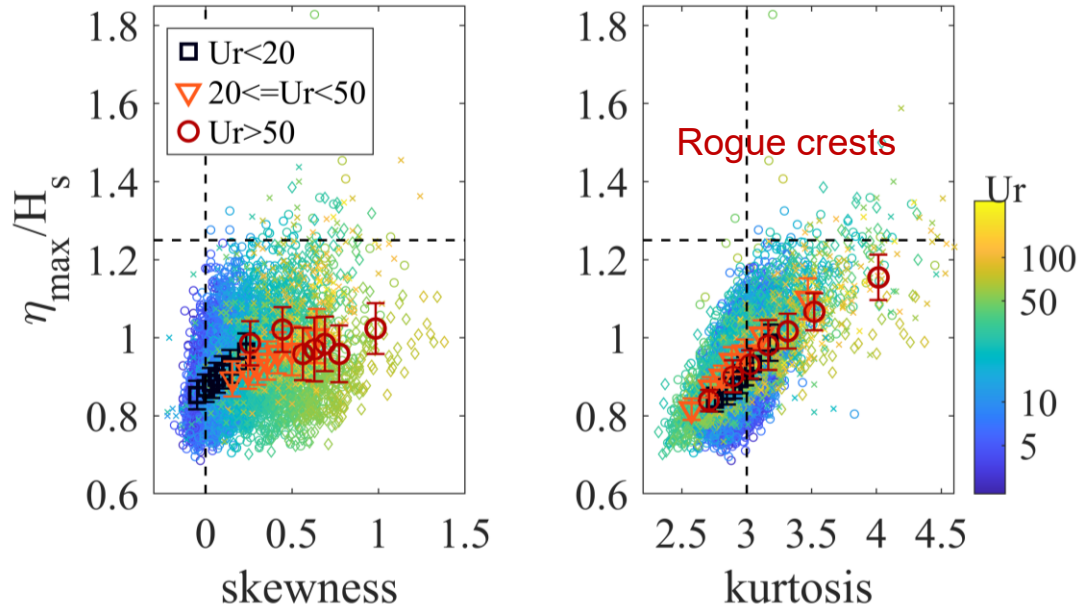
Maximum wave height:

- Decreases with nonlinearity
- Rogue waves extremely rare (~ every few months)

Maximum crest height:

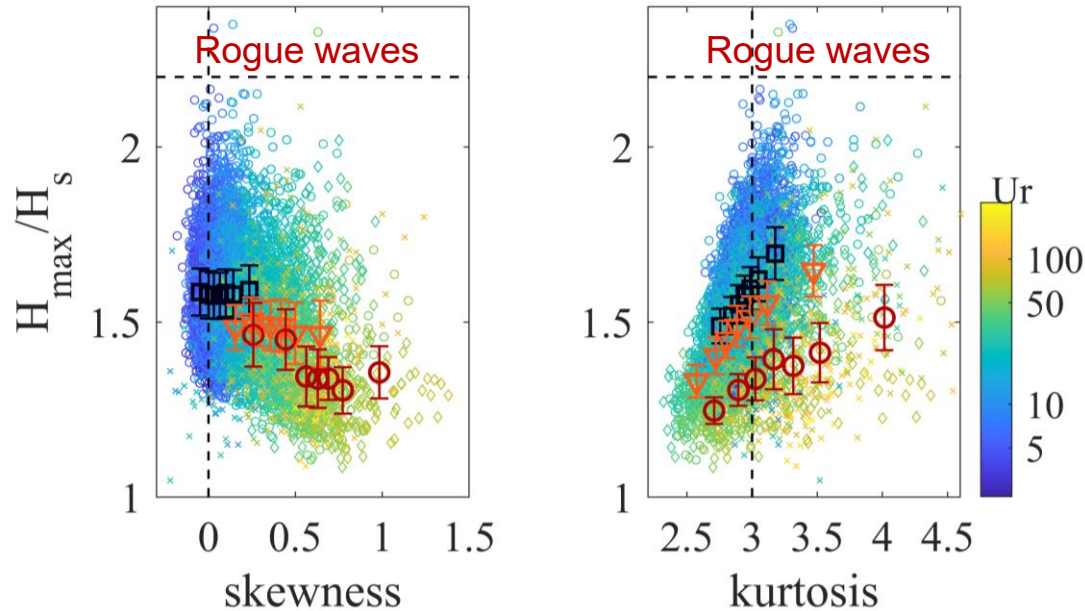
- Increases with nonlinearity
- Rogue crests are common (~ every 10 h)

Maximum crest height – higher moments



- Independent of skewness
- Increases linearly with kurtosis

Maximum wave height – higher moments



- Independent of skewness, but decreases with nonlinearity
- Increases linearly with kurtosis
Dependence decreases with nonlinearity

Summary

- Nearshore fixed locations have wide range of wave nonlinearities
- Results from wave tanks or numerical models are idealized / not representative of real coastal ocean
- Nonlinear surface retrieval from pressure data required
- High crests and small H_w in shallow water
- Skewness – kurtosis: linear dependence
- Excess skewness → little effect on height maxima (crest and waves)
- Excess kurtosis → increased crest maxima, weak increase of wave maxima